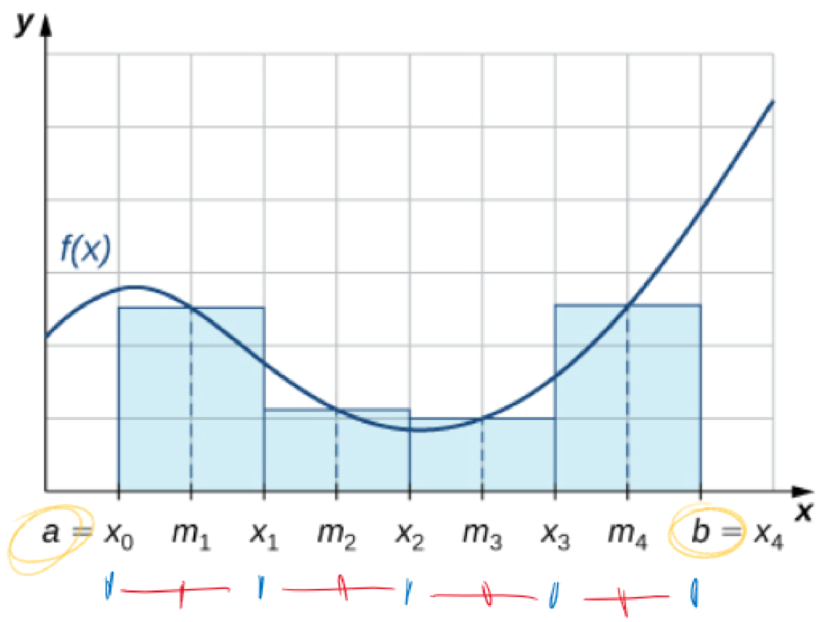
MATH 130A Review: Midpoint Rule

Facts to Know

Integration approximation with the midpoint rule

Picture example



Formal definition

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(m_i) \cdot \Delta x ,$$

where

Δx = width of each partition $= \dfrac{b-a}{n}$

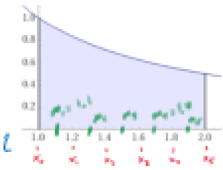
m_i = midpoint of each partition $= a + i \cdot \Delta x - \frac{1}{2}\Delta x$

$= \dfrac{x_i + x_{i-1}}{2}$

Examples

- Use the midpoint rule with $n = 5$ to approximate $\int_1^2 \frac{1}{x}\, dx$

$$\int_{a=1}^{b=2} \frac{1}{x}\, dx \approx \sum_{i=1}^{5} f(m_i)\,\Delta x = f(1.1)\,0.2 + f(1.3)\,0.2 + f(1.5)\,0.2 + f(1.7)\cdot 0.2 + f(1.9)\cdot 0.2 \approx 0.69315$$



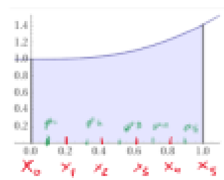
$$\Delta x = \frac{b-a}{n} = \frac{2-1}{5} = \frac{1}{5} = 0.2$$

$$m_i = a + i\cdot\Delta x - \tfrac{1}{2}\Delta x = 1 + i\cdot(0.2) - 0.1$$

$$= \left(\frac{1}{1.1} + \frac{1}{1.3} + \frac{1}{1.5} + \frac{1}{1.7} + \frac{1}{1.9}\right)0.2$$

- Use the midpoint rule with $n = 5$ to approximate $\int_0^1 \sqrt{x^3+1}\, dx$

$$\int_0^1 \sqrt{x^3+1}\, dx \approx \sum_{i=1}^{5} f(m_i)\,\Delta x \approx$$



≈ 1.11145

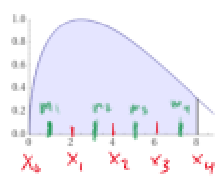
$$\Delta x = \frac{1-0}{5} = 0.2$$

$$m_i = a + i\Delta x - \tfrac{1}{2}\Delta x = 0 + i\,(0.2) - 0.1$$

$$\left(\sqrt{(0.1)^3+1} + \cdots + \sqrt{(0.9)^3+1}\right)0.2$$

- Use the midpoint rule with $n = 4$ to approximate $\int_0^8 \sin\sqrt{x}\, dx$

$$\int_0^8 \sin\sqrt{x}\, dx \qquad \Delta x = \frac{8-0}{4} = 2$$



≈ 5.9979

$$\approx \sum_{i=1}^{4} f(m_i)\,\Delta x$$

$$\approx \left(\sin\sqrt{1} + \sin\sqrt{3} + \sin\sqrt{5} + \sin\sqrt{7}\right)2$$